Appendix I
Recurrence Expression to Evaluate the Green's Function

In this Appendix, we present a recurrence formula to evaluate $\tilde{L}_M(n)$. (M is the number of the interface with conductor strips and n is the number of dielectric layers.)

$$\tilde{L}_M(n) = \tilde{L}'_M(n) + \tilde{L}''_{N-M}(n) - \tilde{g}_{M,M}(n) \quad \text{(A1)}$$

with

$$\tilde{L}'_i(n) = \tilde{g}_{i,i}(n) - \frac{\tilde{g}^2_{i-1,1}(n)}{\tilde{L}'_{i-1}(n)} \quad \text{(A2a)}$$

$$\tilde{L}''_i(n) = \tilde{g}_{N-i,N-i}(n) - \frac{\tilde{g}^2_{N-i,N-i+1}(n)}{\tilde{L}''_{i-1}(n)}, \qquad i = 2,\cdots,N-1. \quad \text{(A2b)}$$

The boundary conditions at lower and upper interfaces for each mode are taken into account by $L'_1(n)$ and $L''_1(n)$ in the following way:

$$\tilde{L}'_1(n) = \tilde{g}_{1,1}(n) - 2pk_n\epsilon^1_{eq} \cdot \left\{\sinh\left(2k_n H^1_{eq}\right)\right\}^{-1} \quad \text{(A3a)}$$

$$\tilde{L}''_1(n) = \tilde{g}_{N-1,N-1}(n), \quad \text{(A3b)}$$

$p = 0$ for odd, even-odd, and odd-odd modes, $p = 1$ for even, even-even, and odd-even modes, k_n is the discrete Fourier variable, and the equivalent heights and permittivities are given by

$$\epsilon^i_{eq} = \sqrt{\epsilon^i_x \cdot \epsilon^i_y} \quad \text{(A4a)}$$

$$H^i_{eq} = H_i \cdot \sqrt{\epsilon^i_x / \epsilon^i_y}. \quad \text{(A4b)}$$

The $\tilde{g}_{i,j}(n)$ are defined as follows:

$$\tilde{g}_{i+1,i}(n) = \tilde{g}_{i,i+1} = -\epsilon_o k_n \epsilon^{i+1}_{eq}\left\{\sinh\left(k_n H^{i+1}_{eq}\right)\right\}^{-1} \quad \text{(A5a)}$$

$$\tilde{g}_{i,i}(n) = \epsilon_o k_n\left\{\epsilon^{i+1}_{eq}\coth\left(k_n H^{i+1}_{eq}\right) + \epsilon^i_{eq}\coth\left(k_n H^i_{eq}\right)\right\}. \quad \text{(A5b)}$$

These expressions are easily evaluated in a digital computer and we can write a subroutine program which considers as inputs the number of dielectric layers and their thickness and permittivity, and provides as output the corresponding Green's function in the spectral domain ($\tilde{G}_M(n) = \tilde{L}_M(n)^{-1}$). Thus, the complexity of the dielectric medium is no longer a difficulty.

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Faster Computation of Z-Matrices for Rectangular Segments in Planar Microstrip Circuits

ABDELAZIZ BENALLA AND K. C. GUPTA

Abstract —**Currently available formulation of Z-matrices for rectangular segments in planar microstrip circuits involves numerical summation of the doubly infinite series in the corresponding Green's function. These computations can be accelerated considerably by using the formulation proposed here which is based on an analytical treatment of one of the summations involved.**

I. Introduction

The planar circuit approach (or two-dimensional circuit approach) has been used for characterization of microstrip components [1]–[5] as well as microstrip antennas [6]–[9]. Microstrip circuits may be treated as planar two-dimensional circuits by using the planar waveguide model [10] for microstrip lines. Segmentation [4], [11] and desegmentation methods [12] used for two-dimensional circuit analysis involve computations of Z-matrices for various planar segments in the circuit. Most common circuit segments have rectangular shapes and the Z-matrix for a rectangular segment is obtained [1], [13] by using a two-dimensional impedance Green's function, which is available as a doubly infinite series involving various modes along the two edges of the rectangular segment. Consequently, computations of Z-matrices based on the currently available formulas [1], [13] involve numerical summation of a doubly infinite series. Because of the slow convergence of these series, a large number of terms (typically 100×100) is needed to obtain sufficiently accurate results. This

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The authors are with the Department of Electrical and Computer Engineering, University of Colorado, Boulder, CO 80309.
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becomes the most time consuming part of planar circuit computations and has discouraged widespread applications of the planar circuit approach.

The present note describes a method leading to faster computation of Z-matrices for rectangular segments. The proposed method involves the summation of a singly infinite series and convergence is much faster. A comparison with the earlier method for a typical case is presented in Section II-D.

II. Computational Method

A. Currently Available Formulation

Calculation of Z-matrices for rectangular segments is based on the Green's function for a rectangle which may be written as [1], [13]

$$G(x_p, y_p | x_q, y_q) = \frac{j\omega\mu d}{ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sigma_m \sigma_n \cdot \frac{\cos(k_x x_p)\cos(k_x x_q)\cos(k_y y_p)}{k_x^2 + k_y^2 - k^2} \cos(k_y y_q). \quad (1)$$

where

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b}$$

$$\sigma_m = \begin{cases} 1, & m = 0 \\ 2, & m \neq 0 \end{cases}$$

$$k^2 = \omega^2 \mu \epsilon_0 \epsilon_r (1 - j\delta)$$

δ = loss tangent of the dielectric.

The length of rectangle is a, its width is b, and height of the substrate is d. The points (x_p, y_p) and (x_q, y_q) denote the location for p and q ports, respectively, which for most of the planar circuit applications are located along the edges of the rectangle. The elements of the Z-matrix for a rectangular segment are obtained from (1) and may be expressed as

$$Z_{pq} = \frac{1}{\eta w_p w_q} \int_{w_p} \int_{w_q} G(x_p, y_p | x_q, y_q)\, dr_p\, dr_q \quad (2)$$

where dr_p and dr_q are incremental distances over the port width w_p and w_q.

$$\eta = \begin{cases} 1, & \text{for microstrip circuit} \\ 2, & \text{for stripline circuits (symmetrical triplate circuits).} \end{cases}$$

For ports oriented along a single direction (x or y) only, the impedance Z_{pq} is given by

$$Z_{pq} = \frac{j\omega\mu d}{\eta ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sigma_m \sigma_n \phi_{mn}(x_p, y_p) \cdot \phi_{mn}(x_q, y_q) / (k_x^2 + k_y^2 - k^2) \quad (3)$$

where for ports oriented along the y-direction

$$\phi_{mn}(x, y) = \cos(k_x x)\cos(k_y y)\,\mathrm{sinc}\left(\frac{k_y w}{2}\right) \quad (4)$$

and for ports oriented along the x-direction

$$\phi_{mn}(x, y) = \cos(k_x x)\cos(k_y y)\,\mathrm{sinc}\left(\frac{k_x w}{2}\right). \quad (5)$$

The function $\mathrm{sinc}(z)$ is defined as $\sin(z)/z$.

Equations (3)–(5) are currently used for computation of Z-matrices for rectangular segments.

B. Proposed Formulation

The doubly infinite series in (3) along with (4) and (5) can be reduced to a singly infinite series by summing the inner sum. The choice of summation over n or m will depend on the relative location of the ports p and q, and also on the aspect ratio of the rectangular segment being considered. Without going into these considerations at this stage, let us call, in general, the index of inner summation as n. The Green's function G given by (1) may be written (separating $n = 0$ term) as

$$G(x_p, y_p | x_q, y_q) = C\left\{ \sum_{m=0}^{\infty} \sigma_m \frac{\cos(k_x x_p)\cos(k_x x_q)}{k_x^2 - k^2} + \sum_{m=0}^{\infty} \sigma_m \cos(k_x x_q)\cos(k_x x_p) S(m) \right\} \quad (6)$$

where $C = j\omega\mu d/ab$ and

$$S(m) = 2\sum_{n=1}^{\infty} \frac{\cos(k_y y_p)\cos(k_y y_q)}{k_x^2 + k_y^2 - k^2}. \quad (7)$$

The summation $S(m)$ may be carried out analytically using trigonometric Fourier series [14] as

$$S(m) = -\frac{b^2}{\pi^2 \alpha_m^2} + \frac{b^2}{2\pi\alpha_m} \frac{ch\,\alpha_m(\pi - x_1) + ch\,\alpha_m(\pi - x_2)}{sh(\alpha_m \pi)} \quad (8)$$

where

$$\alpha_m = \pm \frac{b}{\pi}\sqrt{k_x^2 - k^2}$$

$$x_1 = \frac{\pi(y_< + y_>)}{b} \qquad x_2 = \pi\frac{(y_> - y_<)}{b}$$

$$y_> = \max(y_p, y_q) \qquad y_< = \min(y_p, y_q).$$

Using (8) we can rewrite Green's function G and express it in a symmetrical form by substituting $\alpha_m = \pm j(b/\pi)\gamma_m$. Using l as a dummy variable that could be m or n, we can write Green's function G in the following form

$$G(x_p, y_p | x_q, y_q) = -CF\sum_{l=0}^{\infty} \sigma_l \cos(k_u u_p)\cos(k_u u_q) \cdot \frac{\cos(\gamma_l z_>)\cos(\gamma_l z_<)}{\gamma_l \sin(\gamma_l F)} \quad (9)$$

where

$$F = \begin{cases} b, & l = m \\ a, & l = n \end{cases}$$

$$(u_p, u_q) = \begin{cases} (x_p, x_q), & l = m \\ (y_p, y_q), & l = n \end{cases}$$

$$\gamma_l = \pm\sqrt{k^2 - k_u^2}$$

$$k_u = \begin{cases} \dfrac{m\pi}{a}, & l = m \\ \dfrac{n\pi}{b}, & l = n \end{cases}$$

and

$$(z_>, z_<) = \begin{cases} (y_> - b, y_<), & l = m \\ (x_> - a, x_<), & l = n \end{cases}.$$

Sign of γ_l is chosen such that $Im(\gamma_l)$ is negative.

C. Formulation for Elements of Z-Matrix

Depending on the location of ports p and q, we consider two cases. When ports are located along the edges, they are characterized by their location (x, y) and a width w. For ports located inside the rectangle, we use the concept of effective feed width [15] and characterize these ports also by a location (x, y), an effective width (w) and consider their orientation to be parallel to either x- or y-axis.

Case I: When both ports (p and q) are oriented along the same direction (x or y).

When both the ports are oriented in y direction, the integrations in (2) are with respect to the variable y ($dr_p = dy$ and $dr_q = dy$). In order to keep the $z_>$ and $z_<$ independent of the variable of integration, the inner summation is taken over m (i.e., the dummy variable l is made equal to n) so that $z_> = x_> - a$ and $z_< = x_<$. On the other hand, if both the ports are located along x direction, l is chosen as m and the inner summation is taken over n. These choices ensure the convergence of the series for Z_{pq}. For this case, Z_{pq} is written as

$$Z_{pq} = -CF\frac{1}{\eta}\sum_{l=0}^{\infty}\sigma_l\cos(k_u u_p)\cos(k_u u_q)\cdot\cos(\gamma_l z_>)\cos(\gamma_l z_<)\cdot\frac{\operatorname{sinc}\left(\frac{k_u w_p}{2}\right)\operatorname{sinc}\left(\frac{k_u w_q}{2}\right)}{\gamma_l\sin(\gamma_l F)}. \tag{10}$$

For large values of l, the imaginary part of the arguments of trigonometric functions $\sin(\gamma_l F)$, $\cos(\gamma_l z_>)$, and $\cos(\gamma_l z_<)$ can become very large and give rise to numerical problems. In this situation, the trigonometric functions are replaced by their large argument approximations as

$$\cos(\gamma_l z_>) \simeq \frac{1}{2}e^{-j\gamma_l z_>}$$
$$\cos(\gamma_l z_<) \simeq \frac{1}{2}e^{+j\gamma_l z_<}$$
$$\sin(\gamma_l F) \simeq \frac{1}{2j}e^{+j\gamma_l F}. \tag{11}$$

The sign of γ_l is chosen such that $Im(\gamma_l)$ is negative. The series for Z_{pq} may now be written as

$$Z_{pq} = -CF\frac{1}{\eta}\sum_{l=0}^{L}\sigma_l\cos(k_u u_p)\cos(k_u u_q)\cdot\cos(\gamma_l z_>)\cos(\gamma_l z_<)\cdot\frac{\operatorname{sinc}\left(\frac{k_u w_p}{2}\right)\operatorname{sinc}\left(\frac{k_u w_q}{2}\right)}{\gamma_l\sin(\gamma_l F)} - jCF\frac{1}{\eta}\cdot\sum_{l=L+1}^{\infty}\cos(k_u u_q)\cos(k_u u_p)\operatorname{sinc}\left(\frac{k_u w_p}{2}\right)\cdot\operatorname{sinc}\left(\frac{k_u w_q}{2}\right)\frac{\exp(-j\gamma_l(v_> - v_<))}{\gamma_l} \tag{12}$$

where

$$(v_>, v_<) = \begin{cases}(y_>, y_<) & l = m\\ (x_>, x_<) & l = n\end{cases}.$$

The choice of L becomes a trade-off between fast computation and accuracy. In the numerical example discussed in Section II-D, the algorithm selects L such that the imaginary part of $(\gamma_l F)$ is less than or equal to 500.

Case II: When the two ports (p and q) are oriented in different directions (x and y).

For this case, Z_{pq} given by (2) may be written as

$$Z_{pq} = -CF\frac{1}{\eta}\sum_{l=0}^{\infty}\sigma_l\cos(k_u u_p)\cos(k_u u_q)\cdot\cos(\gamma_l z_<)\cos(\gamma_l z_>)\cdot\operatorname{sinc}\left(\frac{k_u w_i}{2}\right)\operatorname{sinc}\left(\frac{\gamma_l w_j}{2}\right)\Big/(\gamma_l\sin\gamma_l F). \tag{13}$$

If $l = n$, w_i corresponds to the port oriented along y-direction and w_j corresponds to the port along x-direction. On the other hand if $l = m$, w_i is for port along x-direction and w_j for the port along y-direction. Using large argument approximation for trigonometric function; Z_{pq} may be written as

$$Z_{pq} = -CF\frac{1}{\eta}\sum_{l=0}^{L}\sigma_l\cos(k_u u_p)\cos(k_u u_q)\cos(\gamma_l z_<)\cdot\cos(\gamma_l z_>)\frac{\operatorname{sinc}\left(k_u\frac{w_i}{2}\right)\operatorname{sinc}\left(\frac{\gamma_l w_j}{2}\right)}{\gamma_l\sin(\gamma_l F)} - CF\frac{1}{\eta}\sum_{l=L+1}^{\infty}\cos(k_u u_p)\cos(k_u u_q)\cdot\operatorname{sinc}\left(\frac{k_u w_i}{2}\right)\frac{\exp\left(-j\gamma_l\left(v_> - v_< - \frac{w_j}{2}\right)\right)}{\gamma_l^2 w_j}. \tag{14}$$

Choice of l is made by noting that for convergence of the last summation in the above equation, we need

$$(v_> - v_< - w_j/2) > 0. \tag{15}$$

We choose the index of the inner summation so that this condition is satisfied. This condition may be written more explicitly as

$$l = m, \qquad \text{if}\{\max(y_p, y_q) - \min(y_p, y_q) - w_j/2\} > 0 \tag{16}$$

and

$$l = n, \qquad \text{if}\{\max(x_p, x_q) - \min(x_p, x_q) - w_j/2\} > 0. \tag{17}$$

When both of these conditions are satisfied, any choice of l will ensure convergence.

D. Comparison of Two Approaches

A sample comparison of the proposed approach with the existing formulation is illustrated by considering a rectangular segment shown in the inset of Fig. 1. Dimensions chosen are $3\lambda/8 \times 3\lambda/8$ at a frequency of 3 GHz and the substrate is 1/32 inch thick with $\epsilon_r = 2.53$. A nominal loss tangent of 0.001 has been considered. The input impedance of the rectangular segment at the location shown ($x = 0$, $y = \lambda/8$) has been computed both by the existing formula and by the derivation proposed in this paper and the results are presented in Fig. 1. The two plots show percentage error in$|Z_{\text{in}}|$ versus CPU seconds on a Control Data Corporation Cyber 170/720 computer system. The dramatic increase in the computational efficiency offered by the proposed formulation is seen in Fig. 1. The data for these two curves was collected by printing out the CPU time elapsed as a function of number of terms in the summation(s) involved in the computa-

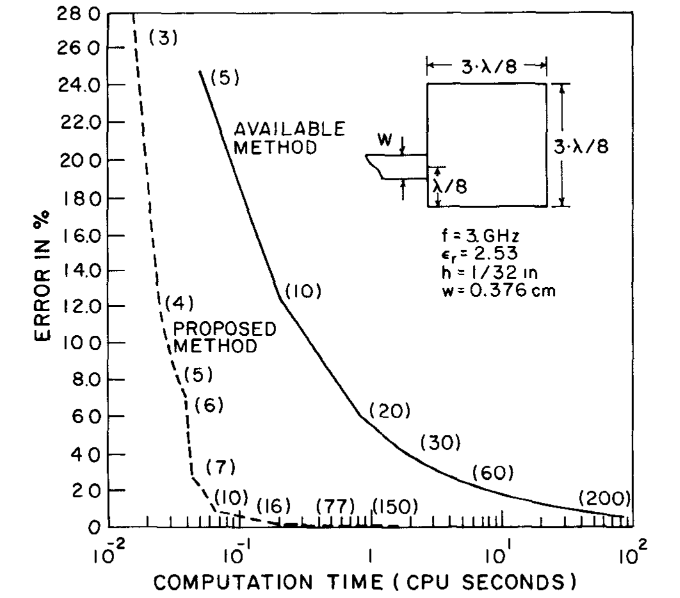

Fig 1 Comparison of two methods for computation of Z-matrix of rectangular planar segments.

tion of input impedance Z_{in}. The number of terms summed up are indicated on two curves. It may be noted that, if the algorithm proposed in this paper is used, the number of terms needed for 1 percent accuracy is 10, while for 0.1 percent accuracy the number of terms needed is 35.

III. Conclusions

A method for faster computations of Z-matrices for rectangular segments in planar microstrip circuits has been presented. As seen by the sample comparison presented, the proposed method yields a dramatic increase in computational efficiency.

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On Gain-Bandwidth Product for Distributed Amplifiers

R. C. BECKER AND J. B. BEYER, SENIOR MEMBER, IEEE

Abstract —**Contours of constant gain-bandwidth product as a function of the gate and drain attenuation factors are presented. Design tradeoffs are established. It is shown that only one design achieves maximum gain-bandwidth, although many possible choices approach this maximum. The curves also lead to the specification of active device parameters when circuit requirements are known.**

I. Introduction

In a previous paper by Beyer *et al.* [1], a graphical design technique was presented which included a curve showing maximum gain-bandwidth product. It will be shown in this paper that the previously presented curve is actually a portion of a more general series of contours of varying gain-bandwidth product. We also show that for the choice of a particular MESFET, there exists only one design for a distributed amplifier that offers maximum gain-bandwidth, however a large number of designs may closely approach this maximum.

In designing microwave-distributed amplifiers, it is usually desirable to attempt to achieve the maximum gain-bandwidth product allowed by the choice of a particular transistor. Because of the nonlinear relationship in a distributed amplifier between gain and bandwidth, their product is influenced by circuit parameters in a complex manner. In this paper, we present a set of curves that augment the graphical techniques presented in [1] and show design tradeoffs clearly.

II. Gain-Bandwidth Contours

Expressing 18 of [1] in terms of -3-dB bandwidth yields

$$A_0 f_{-3\,\mathrm{dB}} = 4KX_{-3\,\mathrm{dB}} f_{\max} \tag{1}$$

where

A = dc gain

$f_{-3\,\mathrm{dB}}$ = half-power frequency

$K = \sqrt{ab}\, e^{-b}$

$X_{-3\,\mathrm{dB}} = f_{-3\,\mathrm{dB}}/f_c$ bandwidth normalized to the line cutoff frequency

$f_{\max}$ = MESFET maximum frequency of oscillation.

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R. C. Becker is with Sperry Corporation, St. Paul, MN.

J. B. Beyer is with the University of Wisconsin-Madison, Department of Electrical and Computer Engineering, Madison, WI 53706.

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